Komplexné čísla $\mathbb{C}$

Prirodzené čísla

$\mathbb{N} = \{1, 2, 3, 4, \dots\}$

$\in$ patrí

Celé čísla $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, 3, 4, \dots\}$

Racionálne čísla $\mathbb{Q} = \{\frac{p}{q}, p \in \mathbb{Z}, q \in \mathbb{N}\}$

Príklad:

$0{,}2 \in \mathbb{Q}$ $\exists p \in \mathbb{Z}, q \in \mathbb{N}$ $p=2$, $q=10$ $\frac{p}{q} = \frac{2}{10} = 0{,}2$

$0{,}31845 \in \mathbb{Q}$ $\exists \dots$ $p = 31845$, $q = 100000$ $\frac{p}{q} = 0{,}31845$

$\sqrt{2}$ – iracionálne číslo

$\nexists p \in \mathbb{Z}, q \in \mathbb{N}$ $\frac{p}{q} = \sqrt{2}$

$\pi = 3{,}14159 2654\dots$

$e = 2{,}718281828 \dots$

Reálne čísla

$$\mathbb{R} = (-\infty, \infty)$$

$x \in \mathbb{R}$

$y \in \mathbb{R}$

$$\left.\begin{array}{l} x+y \\ x-y \\ x \cdot y \\ \dfrac{x}{y} \end{array}\right\} \rightarrow \text{výsledok} \in \mathbb{R}$$

$y \neq 0$

$$\sqrt{x} = y \iff x = y^2$$

$$\sqrt{z} = y$$

$$z = y^2$$

Komplex. čísla $\mathbb{C} = \{ (\underline{x}, \underline{y}),\ x \in \mathbb{R},\ y \in \mathbb{R} \}$

$z_1 = (x_1, y_1)$

$z_2 = (x_2, y_2)$

1. Rovnosť $z_1 = z_2$ ak $x_1 = x_2 \wedge y_1 = y_2$

Príklad $z_1 = (2, 3)$ $\quad z_1 \neq z_2 \quad 2 = 2 \wedge 3 \neq 4$

$z_2 = (2, 4)$

2. Súčin $\mathbb{R}$-čísla a $\mathbb{C}$-čísla

$z = (x, y) \qquad \alpha z = (\alpha x, \alpha y)$

$\alpha \in \mathbb{R}$

Príklad: $z = (2, 3) \qquad \alpha z = (8, 12)$

$\alpha = 4$

Súčin $z_1 = (x_1, y_1)$ $\quad z_1 \cdot z_2 = (x_1, y_1) \cdot (x_2, y_2) =$

$z_2 = (x_2, y_2)$

$$= (\underbrace{x_1 x_2 - y_1 y_2}, \underbrace{x_2 y_1 + x_1 y_2})$$

Príklad:

$z_1 = (2, 1)$ $\quad z_1 \cdot z_2 = (2 \cdot (-1) - 1 \cdot 1, (-1) \cdot 1 + 2 \cdot 1) =$

$z_2 = (-1, 1)$

$$= (-3, 1)$$

Podiel

$z_1 = (x_1, y_1)$

$z_2 = (x_2, y_2)$

$z_2 \neq 0 \quad \wedge \begin{matrix} x_2 \neq 0 \\ y_2 \neq 0 \end{matrix}$

$$\frac{z_1}{z_2} = \left(\frac{x_1 x_2 + y_1 y_2}{x_2^2 + y_2^2}, \frac{-x_1 y_2 + y_1 x_2}{x_2^2 + y_2^2} \right)$$

$z_1 = (0, 1)$

$z_2 = (0, 1)$

$$z_1 \cdot z_2 = z_1^2 = (\overset{x_1}{0}, \overset{y_1}{1}) \cdot (\overset{x_2}{0}, \overset{y_2}{1}) =$$

$$= (0 \cdot 0 - \underbrace{(1)(1)}_{+1}, 0 \cdot (1) + 0 \cdot (1)) =$$

$$= (-1, 0)$$

Ťahák

$$z_1 \cdot z_2 = (x_1, y_1) \cdot (x_2, y_2) =$$

$$= (\underline{x_1 x_2 - y_1 y_2}, \underline{x_2 y_1 + x_1 y_2})$$

$$z_1^2 = (-1, 0) = -1$$

OZN: $(0,1) = i \Leftrightarrow i^2 = (-1, 0) = -1$

ALGEBRICKÝ TVAR $\mathbb{C}$-čísla

$$(x,y) = (x,0) + (0,y) = x(1,0) + y\underbrace{(0,1)}_{i} =$$

$$= x\underbrace{(1,0)}_{\substack{\text{reálna jednotka} \\ \text{(väčšinou sa nepíše)}}} + y\,i = x + \overset{(0,1)}{i}\,y$$

yi – imaginárna jednotka (väčšinou sa píše)

$x, y \in \mathbb{R}$

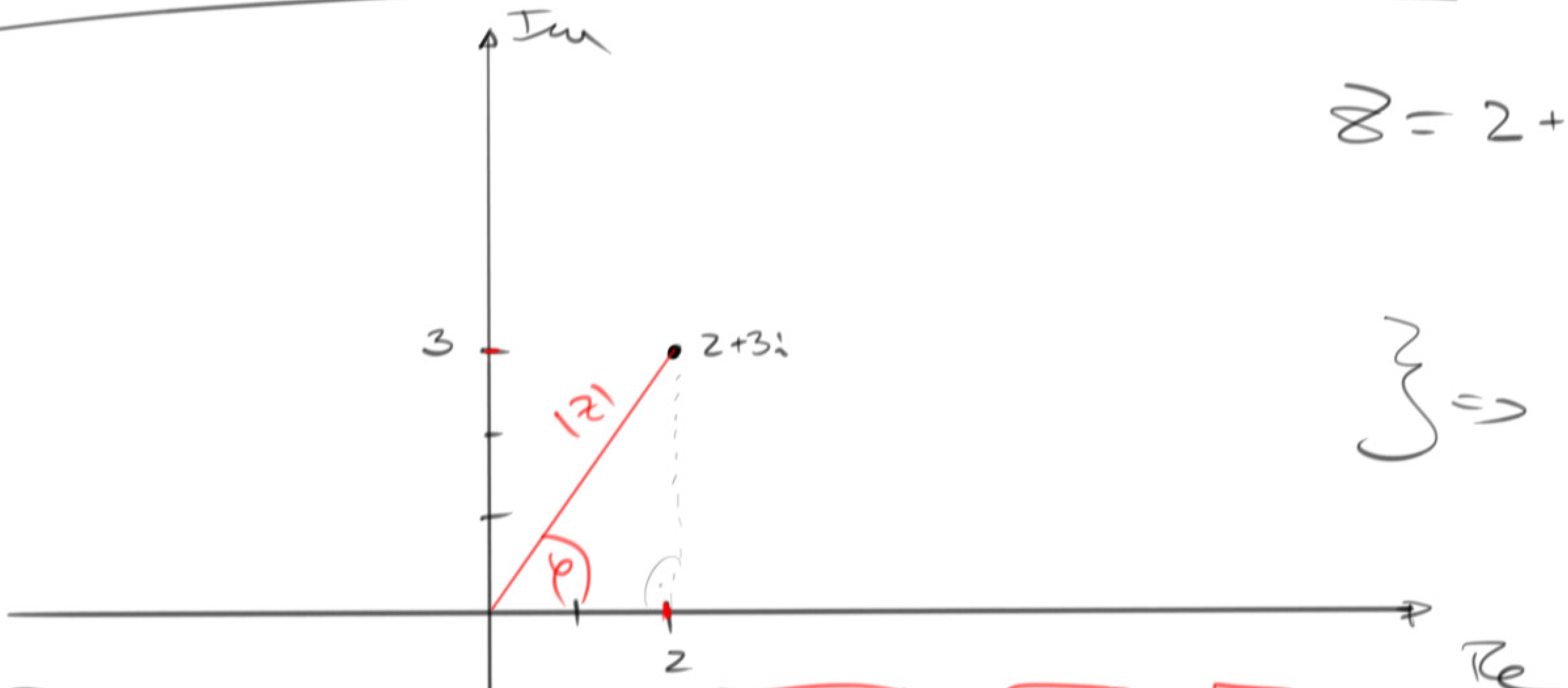
Príklad: $z = (2,3) \overset{A\text{-tvare}}{=} 2 + 3i$

$z = (-1,4) \overset{A}{=} -1 + 4i$

Geometrická reprezentácia $\mathbb{C}$-čísla

$$z = 2 + 3i = (2,3)$$

$\Rightarrow$ Gaussova rovina



$$z = (x,y) = x + iy$$

$$|z| = \sqrt{x^2 + y^2} \in \mathbb{R}$$

$$\left.\begin{aligned} \cos\varphi &= \frac{x}{|z|} \\ \sin\varphi &= \frac{y}{|z|} \end{aligned}\right\} \operatorname{tg}\varphi = \frac{y}{x}$$

$$|z| = \sqrt{2^2 + 3^2} = \sqrt{4+9} = \sqrt{13}$$

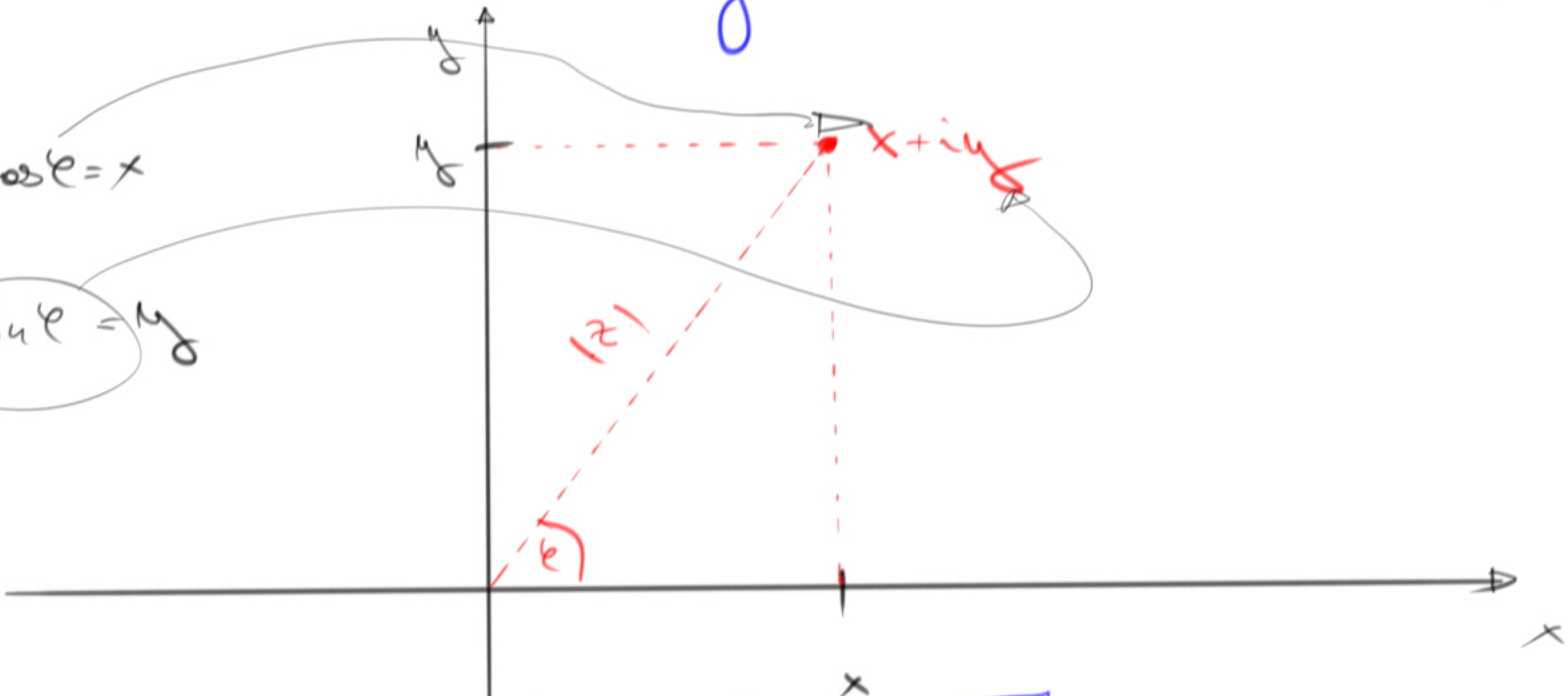
$$\cos\varphi = \frac{2}{\sqrt{13}}$$

$$\sin\varphi = \frac{3}{\sqrt{13}}$$

Goniometrický tvar $\mathbb{C}$-čísla

$$\cos\varphi = \frac{x}{|z|} \iff |z| \cdot \cos\varphi = x$$

$$\sin\varphi = \frac{y}{|z|} \iff |z| \sin\varphi = y$$



$$z = (x,y) = x + iy = \underbrace{|z|}\cos\varphi + i\underbrace{|z|}\sin\varphi = \boxed{|z|(\cos\varphi + i\sin\varphi)}$$

Goniometrický tvar $\mathbb{C}$-čísla

Príklad:

$z = (1,1) = 1 + i$

Transformujte z do goniometrického tvaru

$x = 1 \qquad |z| = ?$

$y = 1 \qquad \varphi = ?$

$|z| = \sqrt{x^2 + y^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$

$$\boxed{\begin{aligned} \cos\varphi &= \frac{x}{|z|} = \frac{1}{\sqrt{2}} \\ \sin\varphi &= \frac{y}{|z|} = \frac{1}{\sqrt{2}} \end{aligned}} \Rightarrow \varphi = \frac{\pi}{4} + k \cdot 2\pi, \; k \in \mathbb{Z}$$

A-tvar

$z = (1,1) = \boxed{1 + i} = |z| \cdot (\cos\varphi + i\sin\varphi) =$

To isté

$= \underbrace{\sqrt{2}\left(\cos\left[\frac{\pi}{4} + k2\pi\right] + i\sin\left[\frac{\pi}{4} + k2\pi\right]\right)}_{\text{G-tvar}}$

$$\boxed{e^{ix} = \cos x + i \sin x}$$

PRAVDA

$$z = (x, y) = x + iy = |z|(\cos\varphi + i\sin\varphi) = \boxed{|z| \cdot e^{i\varphi}}$$

E-tvar

$x = \pi$

$$e^{i\pi} = \cos\pi + i\sin\pi$$

$\cos\pi \to 1$

$i\sin\pi \to 0$

$$e^{i\pi} = 1 \quad / -1$$

$$e^{i\pi} - 1 = 0$$