$\mathbb{C}$ - čísla (pokračovanie)

$\boxed{1.10.2020}$

Príklad:

$z = \sqrt{3} - i$, $\sqrt{3} \to a$, $-i \to b=-1$

transformujte $\mathbb{C}$-čísla z A-tvaru do G-tvaru

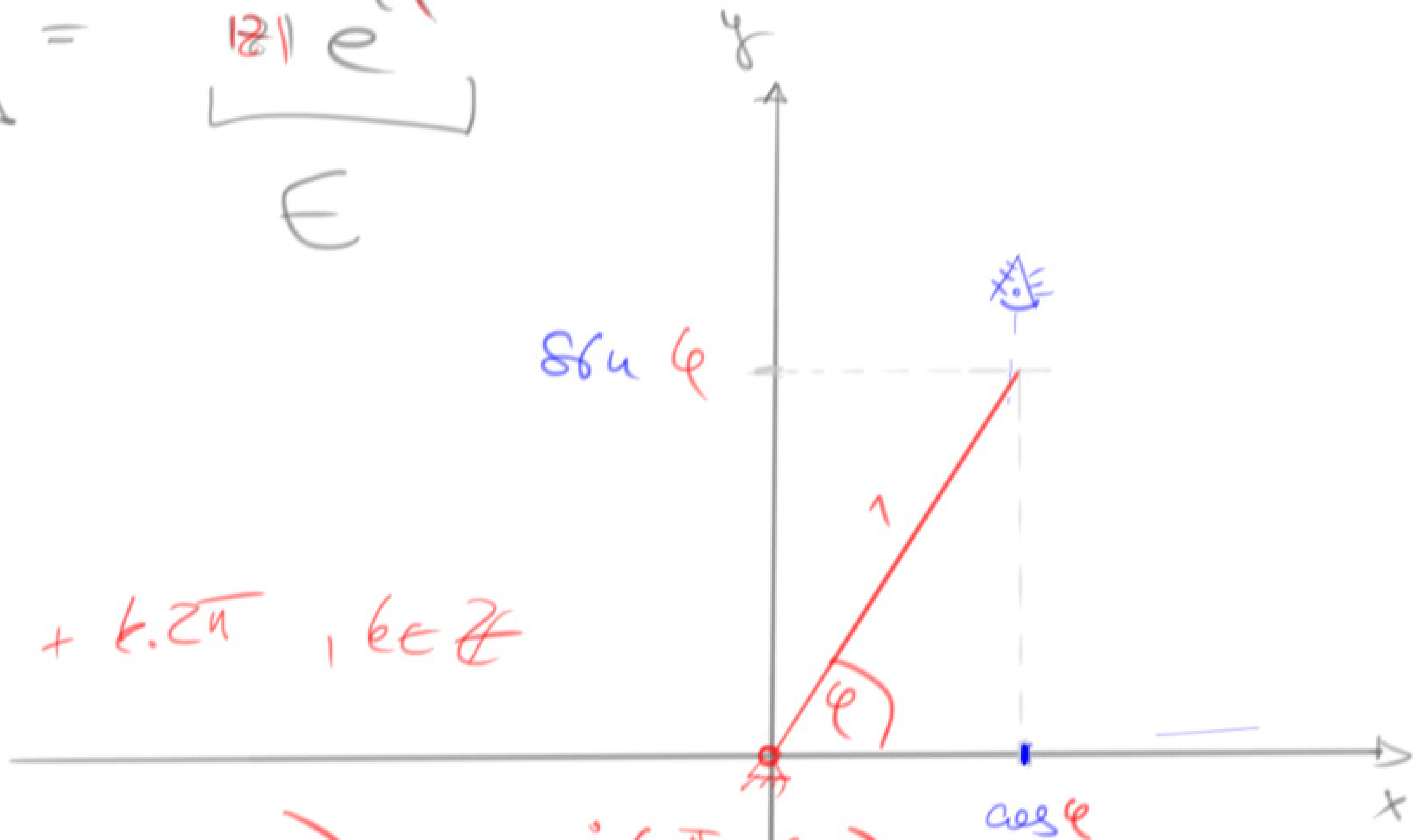
$$z = \underbrace{a + ib = (a, b)}_{A} = \underbrace{|z| \cdot (\cos\varphi + i \sin\varphi)}_{G} = \underbrace{|z| e^{i\varphi}}_{E}$$

$$\sqrt{3} - i = \sqrt{3} + (-1)i = (\underset{a}{\sqrt{3}}, \underset{b}{-1})$$

$$|z| = \sqrt{a^2 + b^2} = \sqrt{(\sqrt{3})^2 + (-1)^2} = \sqrt{3+1} = \sqrt{4} = 2$$

$$\cos\varphi = \frac{x}{|z|} \qquad \sin\varphi = \frac{y}{|z|} \iff \boxed{\begin{aligned}\cos\varphi &= \frac{\sqrt{3}}{2} \\ \sin\varphi &= \frac{-1}{2}\end{aligned}}$$

$$\varphi = -\frac{\pi}{6} + k \cdot 2\pi \ , k \in \mathbb{Z}$$



$$(\sqrt{3}, -1) = \sqrt{3} - i = 2\left(\cos\left[-\frac{\pi}{6} + k2\pi\right] + i \sin\left[-\frac{\pi}{6} + k2\pi\right]\right) = 2e^{i\left(-\frac{\pi}{6} + k2\pi\right)} \ , k \in \mathbb{Z}$$

Mocnina C-čísla

$z = 2+3i$

$$z^2 = (2+3i)^2 = (2+3i)\cdot(2+3i) = 4+6i+6i+9\overset{=-1}{\boxed{i^2}} = \boxed{4+6i+6i-9} =$$

$$= -5+12i = (-5,12)$$

$$z^8 = \underbrace{(2+3i)\cdot(2+3i)}\cdot\underbrace{(2+3i)\cdot(2+3i)}\cdot\underbrace{(2+3i)\cdot(2+3i)}\cdot(2+3i)\cdot(2+3i)$$

$$\underbrace{(-5+12i)\cdot(-5+12i)}\cdot(-5+12i)\cdot(-5+12i)$$

MV

$$\boxed{z = |z|(\cos\varphi + i\sin\varphi), \quad z^n = |z|^n(\cos[n\varphi] + i\sin[n\varphi])}$$

Příklad: $z = 1 + i = (1,1)$

$z^2 = ?$

a) přímo bez MV

$$z^2 = z \cdot z = (1+i)(1+i) = 1 + \underbrace{i + i}_{2i} + \overset{-1}{i^2} = 2i = (0,2)$$

b) přes MV

$$|z| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\left.\begin{aligned} \cos\varphi &= \frac{x}{|z|} \\ \sin\varphi &= \frac{y}{|z|} \end{aligned}\right\} \Rightarrow \left.\begin{aligned} \cos\varphi &= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \\ \sin\varphi &= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \end{aligned}\right\} \Rightarrow \varphi = \frac{\pi}{4} + k2\pi$$

$$(1+i)^2 = \left(\sqrt{2}\left[\cos\left(\tfrac{\pi}{4} + k2\pi\right) + i\sin\left(\tfrac{\pi}{4} + k2\pi\right)\right]\right)^2 \overset{MV}{=} \sqrt{2}^2 \cdot \left(\cos\left[\left(\tfrac{\pi}{4} + k2\pi\right)\cdot 2\right] + i\sin\left[\left(\tfrac{\pi}{4} + k2\pi\right)\cdot 2\right]\right) =$$

$$= 2\cdot\left(\underbrace{\cos\left[\frac{\pi}{2}+k4\pi\right]}_{0} + i\underbrace{\sin\left[\frac{\pi}{2}+k4\pi\right]}_{1}\right) = 2(0+i) = 2i$$

ODMOCNENIE $\mathbb{C}$ - čísla

$$\sqrt[m]{a} = a^{\frac{1}{m}}$$

$$\sqrt[m]{a^n} = a^{\frac{n}{m}}$$

$$\sqrt[n]{z} = z^{\frac{1}{n}} = \sqrt[n]{|z|}\cdot\left[\cos\frac{\varphi}{n} + i\sin\frac{\varphi}{n}\right]$$

$\mathbb{R}$ - číslo

$$\sqrt[n]{z} = \sqrt[n]{|z|}\cdot\left(\cos\frac{\varphi}{n} + i\sin\frac{\varphi}{n}\right)$$

Příklad:

$$\sqrt[4]{1} = \sqrt[4]{(1,0)} = \sqrt[4]{1+0i}$$

$$|z| = |1| = \sqrt{1^2+0^2} = \sqrt{1} = 1$$

$$\cos\varphi = \frac{x}{|z|} \qquad \cos\varphi = \frac{1}{1} = 1$$

$$\sin\varphi = \frac{y}{|z|} \qquad \sin\varphi = \frac{0}{1} = 0$$

$$\left.\begin{array}{l} \cos\varphi = 1 \\ \sin\varphi = 0 \end{array}\right\} \varphi = 0 + k2\pi = k2\pi$$

$$\sqrt[4]{1+0i} = \underbrace{\sqrt[4]{1}}_{=1} \cdot \left(\cos\left[\frac{k2\pi}{\cancel{4}}\right] + i\sin\left[\frac{k2\pi}{\cancel{4}}\right]\right) = \cos k\frac{\pi}{2} + i\sin k\frac{\pi}{2}, \quad k \in \mathbb{Z}$$

a) $k=0$ $\quad \sqrt[4]{1+0i} = \cos\left(0\frac{\pi}{2}\right) + i\operatorname{sen}\left(0\frac{\pi}{2}\right) = \underbrace{\cos 0}_{1} + i\underbrace{\operatorname{sen} 0}_{0} = 1$

b) $k=1$ $\quad \sqrt[4]{1+0i} = \cos\left(1\cdot\frac{\pi}{2}\right) + i\operatorname{sen}\left(1\frac{\pi}{2}\right) = \underbrace{\cos\frac{\pi}{2}}_{0} + i\underbrace{\operatorname{sen}\frac{\pi}{2}}_{1} = \underline{i}$

Se: $\sqrt[4]{1} = i \iff i^4 = 1$

$\underbrace{i\cdot i}_{-1}\cdot\underbrace{i\cdot i}_{-1} = 1$

c) $k=2$ $\quad \sqrt[4]{1+0i} = \cos\left(2\frac{\pi}{2}\right) + i\operatorname{sen}\left(2\frac{\pi}{2}\right) = \underbrace{\cos\pi}_{-1} + i\underbrace{\operatorname{sen}\pi}_{0} = -1$

d) $k=3$ $\quad \sqrt[4]{1+0i} = \cos\left(3\frac{\pi}{2}\right) + i\operatorname{sen}\left(3\frac{\pi}{2}\right) = \underbrace{\cos\left(\frac{3}{2}\pi\right)}_{0} + i\underbrace{\operatorname{sen}\left(\frac{3}{2}\pi\right)}_{-1} = -i$

e) $\cancel{k=4}$ $\quad \cancel{\sqrt[4]{1+0i}} = \cancel{\cos\left(4\frac{\pi}{2}\right) + i\operatorname{sen}\left(4\frac{\pi}{2}\right)} = \cancel{\underbrace{\cos(2\pi)}_{1} + i\underbrace{\operatorname{sen}(2\pi)}_{0}} = \cancel{1}$